Bits and Bytes

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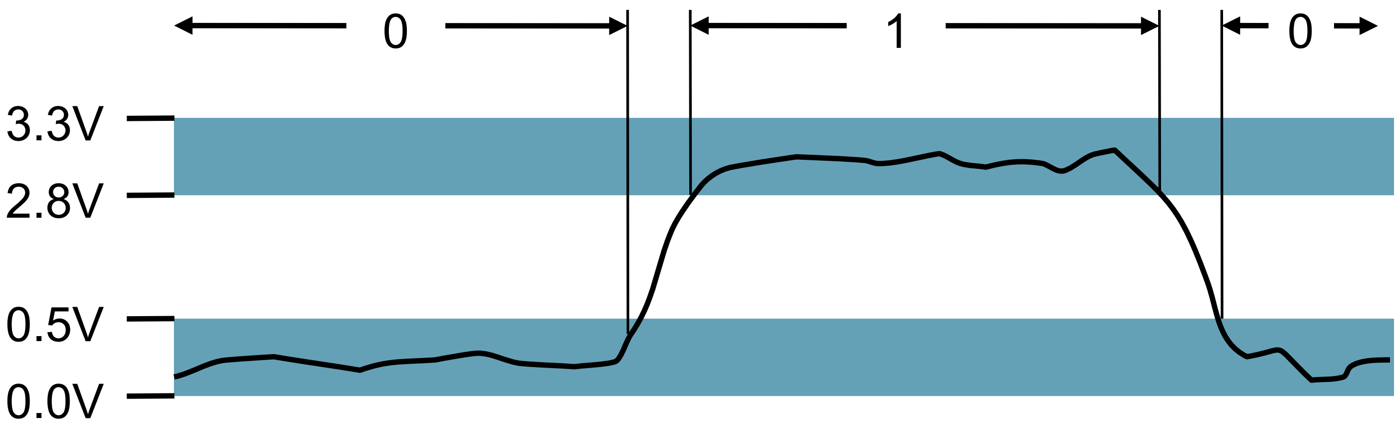
Why don’t computers use Base 10?

- Base 10 number representation
  - “Digit” in many languages also refers to fingers (and toes)
    - Decimal (from latin decimus) means tenth
  - A position numeral system (unlike, say Roman numerals)
  - Natural representation for financial transactions (problems?)
  - Even carries through in scientific notation
- Implementing electronically
  - Hard to store
    - ENIAC (First electronic computer) used 10 vacuum tubes / digit
  - Hard to transmit
    - Need high precision to encode 10 signal levels on single wire
  - Harder to implement digital logic functions
    - Addition, multiplication, etc.

Binary representations

- Base 2 number representation
  - Represent 15213_{10} as 11101101101101_2
  - Represent 1.20_{10} as $1.0011001100110011[0011]..._2$
  - Represent 1.5213×10^4 as $1.1101101101101_2 \times 2^{13}$
- Electronic Implementation
  - Easy to store with bistable elements
  - Reliably transmitted on noisy and inaccurate wires



  - Straightforward implementation of arithmetic functions

Byte-oriented memory organization

- Programs refer to virtual addresses
  - Conceptually very large array of bytes (byte = 8 bits)
  - Actually implemented with hierarchy of different memory types
    - SRAM, DRAM, disk
    - Only allocate for regions actually used by program
  - In Unix and Windows NT, address space private to particular "process"
    - Program being executed
    - Program can manipulate its own data, but not that of others
- Compiler + run-time system control allocation
  - Where different program objects should be stored
  - Multiple mechanisms: static, stack, and heap
  - In any case, all allocation within single virtual address space

How do we represent the address space?

- Hexadecimal notation
  - Base 16 number representation
  - Use characters ‘0’ to ‘9’ and ‘A’ to ‘F’
  - E.g., $\texttt{FA1D37B}_{16}$
    - In C, `0xFA1D37B` or `0xfa1d37b`
  - Each digit unpacks directly to binary
    - `A9` unpacks to `1010 1001`
- Byte = 8 bits
  - Binary: 00000000_2 to 11111111_2
  - Decimal: 0_{10} to 255_{10}
  - Hexadecimal: 00_{16} to FF_{16}

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Checkpoint

Checkpoint

What about Octal?

- Octal notation:
  - Digits 0 through 7, e.g., 7120
    - In C, C++, Java, Javascript…, signaled with leading 0, e.g., 077
    - Source of surprise in things like `new Date(09/11/2011)`
  - Encodes 3 bits at a time
  - Like hex, unpacks directly to binary
  - Unlike hex, no extra digit characters needed
- Used to be a serious competitor to hex
  - Unix **od** command stands for "octal dump"
  - Older architectures had word sizes divisible by 3, e.g., 24, 36, 60
- Octal needed to understand this riddle:
  - Why do programmers confuse Halloween and Christmas?

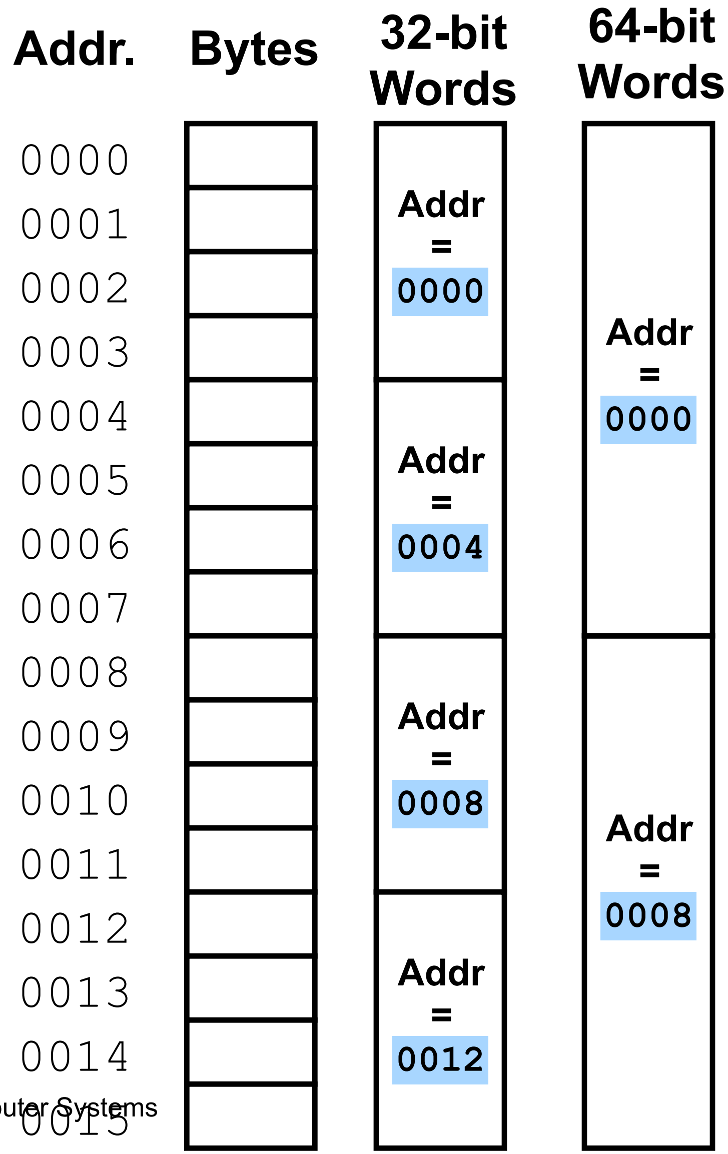
Because 31 OCT = 25 DEC

Machine words

- Machine has "word size"
  - Nominal size of integer-valued data
    - Including addresses
    - A virtual address is encoded by such a word
  - Most current machines are 32 bits (4 bytes)
    - Limits addresses to 4GB
    - Becoming too small for memory-intensive applications
  - High-end systems are 64 bits (8 bytes)
    - Potentially address $\approx 1.8 \times 10^{19}$ bytes
  - Machines support multiple data formats
    - Fractions or multiples of word size
    - Always integral number of bytes

Word-oriented memory organization

- Addresses specify byte locations
  - Address of first byte in word
  - Addresses of successive words differ by 4 (32-bit) or 8 (64-bit)



Data representations

- Sizes of C Objects (in Bytes)

C Data type	Compaq Alpha	Typical 32b	Intel IA32
Int	4	4	4
Long int	8	4	4
Char	1	1	1
Short	2	2	2
Float	4	4	4
Double	8	8	8
Long double	8	8	10/12
Char * (any pointer)	8	4	4

- Portability:
  - Many programmers assume that object declared as *int* can be used to store a pointer
    - *OK for a typical 32-bit machine*
    - *Not for Alpha*

Byte ordering

- How to order bytes within multi-byte word in memory
- Conventions
  - Sun's, Mac's are "Big Endian" machines
    - Least significant byte has highest address (comes last)
  - Alphas, PC's are "Little Endian" machines
    - Least significant byte has lowest address (comes first)
- Example
  - Variable `x` has 4-byte representation `0x01234567`
  - Address given by `&x` is `0x100`

Big Endian

		0x100	0x101	0x102	0x103		
		01	23	45	67		

Little Endian

		0x100	0x101	0x102	0x103		
		67	45	23	01		

Reading byte-reversed Listings

- For most programmers, these issues are invisible
- Except with networking or disassembly
  - Text representation of binary machine code
  - Generated by program that reads the machine code
- Example fragment

Address	Instruction Code	Assembly Rendition	
`8048365:`	`5b`	`pop`	`%ebx`
`8048366:`	`81 c3 ab 12 00 00`	`add`	`$0x12ab,%ebx`
`804836c:`	`83 bb 28 00 00 00 00`	`cmpl`	`$0x0,0x28(%ebx)`

- Deciphering Numbers
  - Value: `0x12ab`
  - Pad to 4 bytes: `0x000012ab`
  - Split into bytes: `00 00 12 ab`
  - Reverse: `ab 12 00 00`

Examining data representations

- Code to print byte representation of data
  - Casting pointer to `unsigned char *` creates byte array

```
typedef unsigned char *pointer;

void show_bytes(pointer start, int len)
{
  int i;
  for (i = 0; i < len; i++)
    printf("0x%p\t0x%.2x\n",
           start+i, start[i]);
  printf("\n");
}
```

Printf directives:

`%p`: **Print pointer**

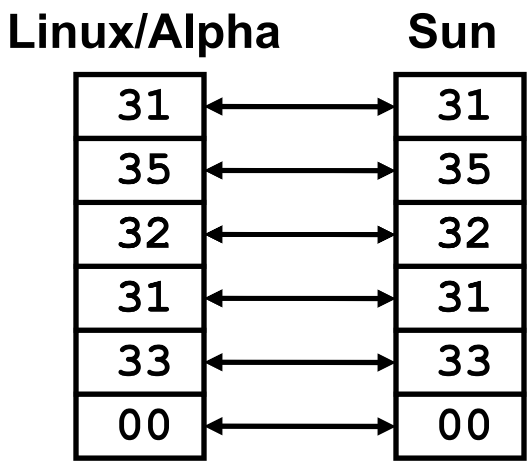
`%x`: **Print Hexadecimal**

Checkpoint

Representing strings in C

- A null-terminated array of characters
  - Final character = 0
- Each character encoded in 7-bit ASCII format
  - Other encodings exist, but uncommon
  - “0” has code 0x30
  - Digit i has code 0x30+i
- Compatibility
  - Byte ordering not an issue
    - Data are single byte quantities
  - Text files generally platform independent
    - Except for different line termination character(s)!

```
char S[6] = "15213";
```



Machine-level code representation

- Encode program as sequence of instructions
  - Each simple operation
    - Arithmetic operation
    - Read or write memory
    - Conditional branch
  - Instructions encoded as bytes
    - Alpha’s, Sun’s, Mac’s use 4 byte instructions
      - Reduced Instruction Set Computer (RISC)
    - PC’s use variable length instructions
      - Complex Instruction Set Computer (CISC)
  - Different instruction types and encodings for different machines
    - Most code not binary compatible
- A fundamental concept:
  Programs are byte sequences too!

Representing instructions

```
int sum(int x, int y)
{
    return x + y;
}
```

- For this example, Alpha & Sun use two 4-byte instructions
  - Use differing numbers of instructions in other cases
- PC uses 7 instructions with lengths 1, 2, and 3 bytes
  - Same for NT and for Linux
  - NT / Linux not fully binary compatible

Alpha sum
00
00
30
42
01
80
FA
6B

Sun sum
81
C3
E0
08
90
02
00
09

PC sum (Linux and NT)
55
89
E5
8B
45
0C
03
45
08
89
EC
5D
C3

Different machines use totally different instructions and encodings

Boolean algebra

- Developed by George Boole in 19th Century
  - Algebraic representation of logic
    - Encode "True" as 1 and "False" as 0

Not ~A

~	
0	1
1	0

And A & B

&	0	1
0	0	0
1	0	1

Or A \| B

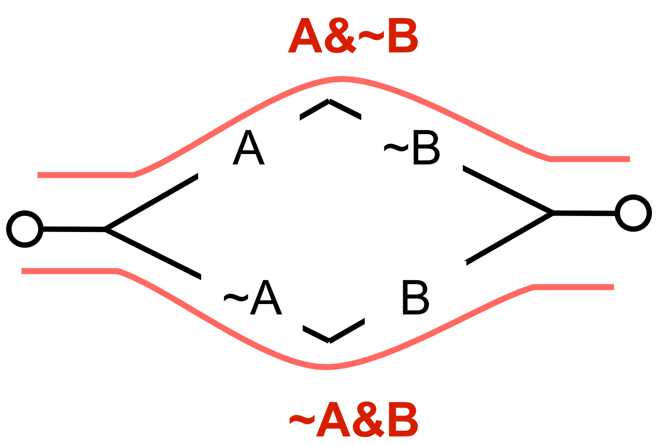
|	0	1
0	0	1
1	1	1

Xor A ^ B

^	0	1
0	0	1
1	1	0

Application of Boolean Algebra

- Applied to Digital Systems by Claude Shannon
  - 1937 MIT Master's Thesis
  - Reason about networks of relay switches
    - Encode closed switch as 1, open switch as 0



Connection when

A&~B | ~A&B

= A^B

Integer Boolean algebra

- Integer Arithmetic

  $\langle Z, +, *, -, 0, 1\rangle$ forms a mathematical structure called "ring"
  - Addition is "sum" operation
  - Multiplication is "product" operation
  - – is additive inverse
  - 0 is identity for sum
  - 1 is identity for product
- Boolean Algebra

  $\langle \{0,1\}, |, \&, \sim, 0, 1\rangle$ forms a mathematical structure called "Boolean algebra"
  - Or is "sum" operation
  - And is "product" operation
  - ~ is "complement" operation (not additive inverse)
  - 0 is identity for sum
  - 1 is identity for product

Boolean Algebra ≈ Integer Ring

Commutativity	$A \mid B = B \mid A$ $A \& B = B \& A$	$A + B = B + A$ $A * B = B * A$
Associativity	$(A \mid B) \mid C = A \mid (B \mid C)$ $(A \& B) \& C = A \& (B \& C)$	$(A + B) + C = A + (B + C)$ $(A * B) * C = A * (B * C)$
Product distributes over sum	$A \& (B \mid C) = (A \& B) \mid (A \& C)$	$A * (B + C) = A * B + A * C$
Sum and product identities	$A \mid 0 = A$ $A \& 1 = A$	$A + 0 = A$ $A * 1 = A$
Zero is product annihilator	$A \& 0 = 0$	$A * 0 = 0$
Cancellation of negation	$\sim(\sim A) = A$	$-(-A) = A$

Boolean Algebra $\neq$ Integer Ring

Boolean, not Ring: Sum distributes over product	$A \mid (B \,\&\, C) = (A \mid B) \,\&\, (A \mid C)$	$A + (B * C) \neq (A + B) * (B + C)$
Boolean, not Ring: Idempotency	$A \mid A = A$ $A \,\&\, A = A$	$A + A \neq A$ $A * A \neq A$
Boolean, not Ring: Absorption	$A \mid (A \,\&\, B) = A$ $A \,\&\, (A \mid B) = A$	$A + (A * B) \neq A$ $A * (A + B) \neq A$
Boolean, not Ring: Laws of Complements	$A \mid {\sim}A = 1$	$A + -A \neq 1$
Ring, not Boolean: Every element has additive inverse	$A \mid {\sim}A \neq 0$	$A + -A = 0$

Properties of & and ^

- Boolean ring

  $\langle\{0,1\}, \wedge, \&, \mathrm{I}, 0, 1\rangle$

  – Identical to integers mod 2
  – I is identity operation: I (A) = A
    - A ^ A = 0
- Property: Boolean ring
  – Commutative sum A ^ B = B ^ A
  – Commutative product A & B = B & A
  – Associative sum (A ^ B) ^ C = A ^ (B ^ C)
  – Associative product (A & B) & C = A & (B & C)
  – Prod. over sum A & (B ^ C) = (A & B) ^ (B & C)
  – 0 is sum identity A ^ 0 = A
  – 1 is prod. identity A & 1 = A
  – 0 is product annihilator A & 0 = 0
  – Additive inverse A ^ A = 0

Checkpoint

Relations between operations

- DeMorgan's Laws
  - Express in terms of |, and vice-versa
    - A & B = ~(~A | ~B)
      - A and B are true if and only if neither A nor B is false
    - A | B = ~(~A & ~B)
      - A or B are true if and only if A and B are not both false
- Exclusive-Or using Inclusive Or
    - A ^ B = (~A & B) | (A & ~B)
      - Exactly one of A and B is true
    - A ^ B = (A | B) ~(A & B)
      - Either A is true, or B is true, but not both

General Boolean algebras

- We can extend the four Boolean operations to also operate on bit vectors
  - Operations applied bitwise

```
  01101001     01101001     01101001
& 01010101   | 01010101   ^ 01010101   ~ 01010101
  01000001     01111101     00111100     10101010
```

- All of the Properties of Boolean Algebra Apply
- Resulting algebras:
  - Boolean algebra: ⟨{0,1}(w), |, &, ~, 0(w), 1(w)⟩
  - Ring: ⟨{0,1}(w), ^, &, I, 0(w), 1(w)⟩

Representing manipulating sets

- Useful application of bit vectors – represent finite sets
- Representation
  - Width w bit vector represents subsets of {0, …, w–1}
  - $a_j = 1$ if $j \in A$
    - 01101001 represents { 0, 3, 5, 6 }
    - 01010101 represents { 0, 2, 4, 6 }

0	1	1	0	1	0	0	1
7	6	5	4	3	2	1	0

- Operations
  - & Intersection 01000001 { 0, 6 }
  - | Union 01111101 { 0, 2, 3, 4, 5, 6 }
  - ^ Symmetric difference 00111100 { 2, 3, 4, 5 }
  - ~Complement 10101010 { 1, 3, 5, 7 }

Bit-level operations in C

- Operations &, |, ~, ^ available in C
  - Apply to any “integral” data type
    - `long`, `int`, `short`, `char`
  - View arguments as bit vectors
  - Arguments applied bit-wise
- Examples (Char data type)
  - `~0x41 --> 0xBE`
    $\sim 01000001_2$ --> 10111110_2
  - `~0x00 --> 0xFF`
    $\sim 00000000_2$ --> 11111111_2
  - `0x69 & 0x55 --> 0x41`
    01101001_2 01010101_2 --> 01000001_2
  - `0x69 | 0x55 --> 0x7D`
    01101001_2 | 01010101_2 --> 01111101_2

Logic operations in C – not quite the same

- Contrast to logical operators
  - &&, ||, !
    - View 0 as “False”
    - Anything nonzero as “True”
    - Always return 0 or 1
    - Early termination (if you can answer looking at first argument, you are done)
- Examples (char data type)
  - !0x41 --> 0x00
  - !0x00 --> 0x01
  - !!0x41 --> 0x01

  - 0x69 && 0x55 --> 0x01
  - 0x69 || 0x55 --> 0x01

Shift operations

- Left shift: x << y
  - Shift bit-vector x left y positions
    - Throw away extra bits on left
    - Fill with 0's on right
- Right shift: x << y
  - Shift bit-vector x right y positions
    - Throw away extra bits on right
  - Logical shift
    - Fill with 0's on left
  - Arithmetic shift
    - Replicate most significant bit on right
    - Useful with two's complement integer representation

Argument x	01100010
<< 3	00010000
Log. >> 2	00011000
Arith. >> 2	00011000

Argument x	10100010
<< 3	00010000
Log. >> 2	00101000
Arith. >> 2	11101000

Main points

- It's all about bits & bytes
  - Numbers
  - Programs
  - Text
- Different machines follow different conventions
  - Word size
  - Byte ordering
  - Representations
- Boolean algebra is mathematical basis
  - Basic form encodes "false" as 0, "true" as 1
  - General form like bit-level operations in C
    - Good for representing manipulating sets